M.Sc. - II (Mathematics) (New CBCS Pattern) Semester-III

PSCMTH13 - Paper-III : Mathematical Methods

P. Pages : 3 **GUG/S/25/13757**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) If $f(t) \in P'(R)$ and $f(t) \in A_1(R)$, then for all $x \in R$, show that **10**

$$\frac{1}{\pi}\int_0^{\infty} dy \int_{-\infty}^{+\infty} f(t)\cos\left[y(x-t)\right]dt = \frac{1}{2}\left[f(x+0)+f(x-0)\right]$$

b) State and prove the linearity properties of Fourier transforms. **10**

OR

c) State and obtain the Parseval's relations for Fourier transforms. **10**

d) Evaluate $F_C\left[\left(a^2-x^2\right)^{\nu-\frac{1}{2}} H(a-x); x \to \xi\right]$ **10**

UNIT – II

2. a) Solve the wave equation **10**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 v}{\partial t^2},\ 0 \le x \le a,\ t > 0$$

Satisfying the boundary condition $u(0,t) = u(a,t) = 0, t > 0$ and the initial conditions $u(x,0) = \frac{4b}{a^2}x(a-x),\ \frac{\partial u(x,0)}{\partial t} = 0,\ 0 \le x \le a$ to determine the displacement u(x, t).

b) Let f(x) be continuous and $f'(x)$ be sectionally continuous on the interval $0 \le x \le a$, then prove that **10**

i) $\bar{f}_c\left[f'(x); x \to n\right] = (-1)^n f(a) - f(0) + \frac{n\pi}{a}\bar{f}_s(n), n \in Z^*$

ii) $\bar{f}_s\left[f'(x); x \to n\right] = \frac{-n\pi}{a}\bar{f}_c(n), n \in N$

OR

c) The end points of a solid bounded by $x = 0$ and $x = \pi$ are maintained at temperatures $u(0,t) = 1$, $u(\pi, t) = 3$, where $u(x,t)$ represents its temperature at any point of it at any time t. Initially, the solid was held at 1 unit temperature with its surfaces were insulated. Find the temperature distribution $u(x,t)$ of the solid, given that $u_{xx}(x,t) = u_t(x,t)$. **10**

d) The transverse displacement of an elastic membrane u(x, y, t) satisfies the PDE **10**
$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, under the boundary conditions u = 0 on the boundary,
$u = f(x,y)$, $u_t = g(x,y)$ at t = 0. Find the displacement $u(x,y,t)$ after utilizing the finite Fourier transform.

UNIT – III

3. a) If f(t) is of some exponential order for large t & is piecewise continuous over $0 \le t \le \infty$, then prove that Laplace transform of f(t) exists. **10**

b) Evaluate **10**

i) $L\left[t\,H(t-1); t \to P\right]$ &

ii) $L\left[H(t-1)\cos at;\ t \to P\right]$.

OR

c) Prove that $L\left[e^{-\frac{t^2}{4}}; t \to P\right] = \sqrt{\pi}\, e^{P^2}\, Erfc(P)$ **10**

d) Evaluate $L\left[\frac{\sin at}{t}\right]$. Does $L\left[\frac{\cos at}{t}\right]$ exist ? **10**

UNIT – IV

4. a) If $\overline{f}_n(\xi)$ be Hankel transform of f(r) of order n, then prove that Hankel transform of f ’(r) of order η is given by $H_n\left[f'(r);\xi\right] = \frac{\xi}{2n}\left[(n-1)\,\overline{f}_{n+1}(\xi) - (n+1)\,\overline{f}_{n-1}(\xi)\right]$, $n \ge 1$ **10**

& $H_1\left[f'(r);\xi\right] = -\xi \overline{f}_0(\xi)$

provided $\left[r\,f(r)\right]$ Vanishes as $r \to \infty$ and as $r \to 0$.

b) State & prove the Parseval theorem. **10**

OR

c) If $f^*(s)$ and $g^*(s)$ be Mellin Transforms of $f(x)$ and $g(x)$ respectively, then prove that **10**

$$M\left[f(x)\,g(x); x \to s\right] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} f^*(z)\,g^*(s-z)\,dz.$$

d) Evaluate Mellin transform of **10**

i) $f(x) = e^{-nx}$

ii) $f(x) = (1+x)^{-a}$

iii) $f(x) = \left(1+x^a\right)^{-b}$, where $0 < \text{Re } s < \text{Re}(ab)$

UNIT – V

5. a) If $F[f(x); x \to \xi] = F(\xi)$, then show that $F[f(ax); x \to \xi] = \frac{1}{a} F\left(\frac{\xi}{a}\right)$. **5**

b) Let $f(x)$ and $f'(x)$ be continuous and $f''(x)$ be sectionally continuous in $0 \le x \le a$ then show that $\bar{f}_c[f''(x); n] = -f'(0) + (-1)^n f'(a) - \frac{n^2\pi^2}{a^2} \bar{f}_c(n)$. **5**

c) Evaluate $L^{-1}\left[\frac{p+s}{(p+1)\left(p^2+1\right)}\right]$. **5**

d) State and prove the linearity properly of Hankel transform. **5**

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